Forum for Electromagnetic Research Methods and Application Technologies (FERMAT)

Time Domain Method of Moment (MoM) and Physical Optics (PO) Hybrid Method with Reduced Iterations

Zicong Mei [1]
[1] Lorentz Solution, Inc., Santa Clara, CA
(Email: zmei@syr.edu)

***Abstract*— The frequency domain hybrid Method of Moment (MoM) and Physical Optics (PO) usually requires a iteration between MoM and PO region. This is a reasonable choice for frequency domain because most of the computation cost is in factorizing the MoM matrix and while the cost of back substitution to the matrix equation is negligible. But it is not the case for the time domain version. In time domain method, the back substitution is the major bottle neck for computation time because it is done for each time step for marching-on-in-time or order step for marching-on-in-degree. So the iteration between MoM and PO region become costly because it requires to compute all the time or order steps for each iteration. In this paper, I presented a method that can avoid this iteration and thus speed up the computation speed for several times.**

***Index Terms*— Method of Moments (MoM), time domain electric field integral equation (TD-EFIE), physical optics (PO).**

I. INTRODUCTION

The Method of Moment (MoM) is a widely used technique to solve electromagnetic field problems.[1][2] But as the size of the problem increases, the matrix size and the computation time will scale as the square and cube of the unknown numbers, respectively. Therefore, solution of electrically large problems using MoM becomes quite challenging. However, in many practical cases, these large objects are metallic and the surfaces are relatively smooth. So it is reasonable to hybridize a MoM methodology with asymptotic techniques.[3]-[8]

In this paper, a faster time domain MoM and Physical Optics (PO) hybrid method is proposed. This new method based one the traditional theory but can solve the problem in a much faster manner by reducing the iterations.

For traditional hybrid MoM-PO method, the problem of interest can be divided into two regions. One is the MoM region and the other is PO region. Assuming the PO current to be zero, the currents in the MoM region are solved using the time domain electric field integral equations (TD-EFIE), which was already studied by extensive researchers[9]-[17]. Then the scattered field from MoM region induces a current on the PO region and which can be calculated by PO method. Hence, the PO current is known. Treating this new PO current as an additional source for MoM problem, the original MoM current can be updated. By repeating this iteration, one converges towards the final solution.

But this iteration takes a significant amount of time and I discovered a method that can avoid this iteration but can still obtain the same result at no additional cost.

In section 2, I will briefly describe the traditional time domain MoM-PO method. In Section 3, the proposed new method will be described. The cost of the new method is analyzed in Section 4. And finally numerical examples will be provided in Section 5.

II. TRADITIONAL SOLUTION TO THE HYBRID METHOD OF TIME DOMAIN MOM AND PO

For a hybrid MoM and PO problem, the structure can be divided into two regions. One is the MoM region governed by MoM formulas and the other is the PO region. For simplicity, we discuss only perfect electric conductor (PEC) for in this paper.

For the MoM region, the boundary condition on PEC surface is the tangential electric field is zero.

$$\left[\mathbf{E}^{MoM}(\mathbf{r},t)+\mathbf{E}^{PO}(\mathbf{r},t)+\mathbf{E}^{exciatation}(\mathbf{r},t)\right]_{\tan}=0 \tag{1}$$

where $\mathbf{E}^{MoM}(\mathbf{r},t)$, and $\mathbf{E}^{PO}(\mathbf{r},t)$, $\mathbf{E}^{exciatation}(\mathbf{r},t)$ denote the scattered wave from the MoM region, the scattered wave from the PO region, and the excitation wave, respectively. And the subscript *tan* implies the tangential components of the fields. The scattered fields $\mathbf{E}(\mathbf{r},t)$ can be obtained from the current. Suppose we know the current in the PO region, then the scattered wave from the PO region $\mathbf{E}^{PO}(\mathbf{r},t)$ can be treated as a special form of excitation wave and the current in the MoM region can be solved similar to previous MoM methods. [9][10]

This use of this work is restricted solely for academic purposes. The author of this work owns the copyright and no reproduction in any form is permitted without written permission by the author.

If the current in the MoM region is already known, the current over the PEC PO region is given by the boundary condition of Eq. (2) and (3). In the PO region, the mutual interaction with PO current at other positions is neglected. The total magnetic field can be substituted by two times of the wave incident to the PO surface at the lit region S_{lit} and 0 in the shadowed region S_{shadow}.

$$\mathbf{J}^{PO}(\mathbf{r},t) = \hat{\mathbf{n}} \times \mathbf{H}(\mathbf{r},t) \tag{2}$$

$$\mathbf{J}^{PO}(\mathbf{r},t) = \begin{cases} 2\hat{\mathbf{n}} \times \left(\mathbf{H}^{exciatation}(\mathbf{r},t) + \mathbf{H}^{MoM}(\mathbf{r},t) \right) & \mathbf{r} \in S_{lit} \\ 0 & \mathbf{r} \in S_{shadow} \end{cases}$$

(3)

where $\mathbf{H}^{exciatation}$ is the excitation magnetic field and the $\mathbf{H}^{MoM}$ is the scattered magnetic field from the MoM current.

The hybrid MoM-PO methodology can now be outlined using an iterative method.

1. We first let the initial guess for the PO unknown current to be zero, i.e., $\mathbf{J}_{(0)}^{PO} = 0$
2. The MoM unknown coefficients at the *k*-th iteration $\mathbf{J}_{(k-1)}^{MoM}$ can be evaluated by the PO coefficients of the previous iteration $\mathbf{J}_{(k-1)}^{PO}$.
3. The PO unknown coefficients at the *k*-th iteration $\mathbf{J}_{(k)}^{PO}$ can be evaluated by the MoM coefficients of the previous iteration $\mathbf{J}_{(k)}^{MoM}$.
4. If the results have not demonstrated convergence, then go back to step 2.

In this method, we have an intention in solving the coupling of the coupling between MoM and PO. But in each time-domain MoM solution, it also involves many time step or order step iterations to obtain the MoM solution from an excitation wave. So the total iteration number will be the numbers of these two types of iteration multiply up.

III. A METHOD TO REDUCE THE ITERATION NUMBER

The previous method simulates the wave bounce back and forth in physics point of view and easy to formula. But it needs to solve the MoM problem for multiple times. This will not be a problem if this is frequency domain problem because the MoM matrix is the same for all iterations and thus the matrix factorization only need to be performed once and back substitution is fast.

However, for time domain problem, the computation cost is different. In many cases, the major cost in solving MoM problem is not the matrix factorization but the backward substitution because it needs to be performed for each time step for marching-on-in-time solver or order step for marching-on-in-degree solver. So reducing the iteration number will significantly speed up the computation.

If we look at the problem in a matrix format, the iteration between the MoM and PO coupling can be removed. As described in previous section, the effect of PO current is considered as a kind of excitation wave.

$$\left[Z^{MoM}\right]\left[J^{MoM}\right] = \left[V^{exciatation-MoM}\right] + \left[Z^{PO-MoM}\right]\left[J^{PO}\right] \tag{4}$$

in which Z^{MoM} is MoM impedance matrix, J^{MoM} the unknown vector of the MoM currents, J^{PO} is the vector of the PO current and $Z^{PO\text{-}MoM}$ is the impedance matrix of the effect of PO currents, and $V^{excitation\text{-}MoM}$ is the excitation wave.

The PO current is calculated by the scattered wave from the MoM current and the excitation wave. The difference here is that it is not necessary to compute a matrix equation in order to obtain the PO current when the excitation is known.

$$\left[J^{PO}\right] = \left[V^{exciatation-PO}\right] + \left[Z^{MoM-PO}\right]\left[J^{MoM}\right] \tag{5}$$

in which $V^{excitation\text{-}PO}$ is the excitation wave on the MoM region and $Z^{MoM\text{-}PO}$ is the impedance matrix of the effect of MoM currents.

In the traditional method, researchers solve Eq. (4) and (5) in an iterative way described in previous section. But when we write it in a matrix format, it is clear that Eq. (5) can be substitute into Eq. (4) and one can obtain the following equation.

$$\begin{aligned} &\left(\left[Z^{MoM}\right] - \left[Z^{PO-MoM}\right]\left[Z^{MoM-PO}\right]\right)\left[J^{MoM}\right] \\ &\quad = \left[V^{exciatation-MoM}\right] + \left[Z^{PO-MoM}\right]\left[V^{exciatation-PO}\right] \end{aligned} \tag{6}$$

The J^{MoM} in Eq. (6) can be directly solved without any iteration. And it implies a faster simulation. After J^{MoM} is known, J^{PO} can be easily computed by Eq. (5).

IV. MATRIX FILLING TECHNIQUE NOT TO INCREASE MEMORY USAGE

A major difficulty in this equation in the memory space needed. In the traditional method, the matrix requires a storage space is Z^{MoM} and the size is N^{MoM}-by-N^{MoM}, in which N^{MoM} is the number of unknowns in the MoM region.

But in this formula, we need two more matrixes $Z^{PO\text{-}MoM}$ and $Z^{MoM\text{-}PO}$. , which are N^{PO}-by-N^{MoM} and N^{MoM}-by-N^{PO}, respectively, in which N^{PO} is the number of unknowns in the PO region. The size of N^{PO} is usually much larger than N^{MoM}. If we need to store these two matrixes, it may require unaffordable amount of memory space.

Luckily, there is a filling method that can avoid this expensive memory usage. We can calculate the *i*-th column of $Z^{PO\text{-}MoM}$ and the *i*-th row of $Z^{MoM\text{-}PO}$. One can multiplies this column to this row and add the result to the matrix Z^{MoM}. After doing this for all rows and columns, the multiplication of two matrixes of $Z^{PO\text{-}MoM}$ and $Z^{MoM\text{-}PO}$ is done and no additional memory space is required. What is more, there are numerical packages such as MKL that can perform this operation in a very fast way.

V. NUMERICAL EXAMPLES

Sample numerical result is presented to illustrate the versatility of this method. In this work, all the structures are excited using a T-pulse.[18]-[22] A T-pulse is a strictly time-limited pulse with most of its energy concentrated in a narrow band. Hence the waveform by definition is causal.

Example 1 deals with a *z*-polarized bicone antenna with reflector as shown in Figure 1. The bicone has a radius of 0.2m and its length is 2.1m. It also has a wire of 0.1m connecting the two cones in the middle. The wire has a length of 0.1m and radius of 1mm. The excitation locates at the center of the wire. A 2m-by-2m square reflector is placed 2m away from the bicone. The MoM region contains only this bicone with 750 unknowns and the PO region contains the reflector with 2094 unknowns. A T-pulse with duration of 23.33 nanosecond, which implies a bandwidth of 300 MHz is radiated by the bicone.

The current at the center of the wire is calculated with both traditional and proposed MoM-PO hybrid methods and plotted in Figure 3. It can be seen that the proposed method matches the traditional method well. Also the memory and time usage are compared in Table 1. It can be seen that the memory usage is not changed but the computation time is speeded up for 3.11 times. That is because the tradition method needs three iterations between MoM and PO current to converge for this case.

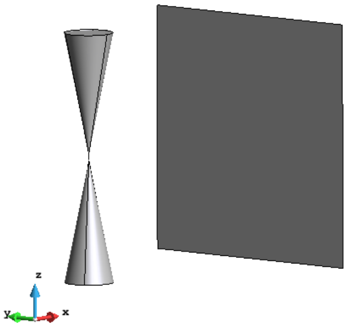

Figure 1. A bicone antenna with square reflector.

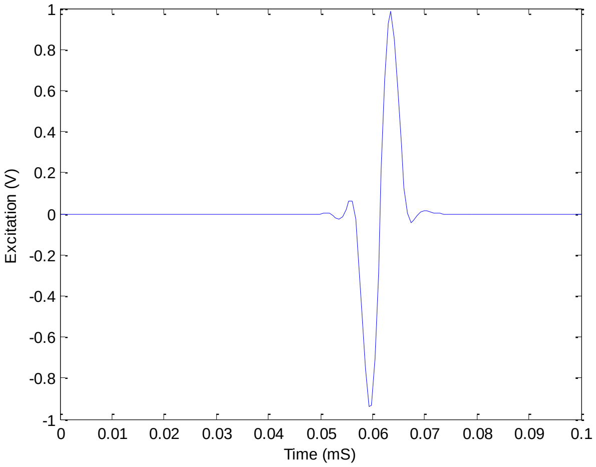

Figure 2. The excitation pulse..

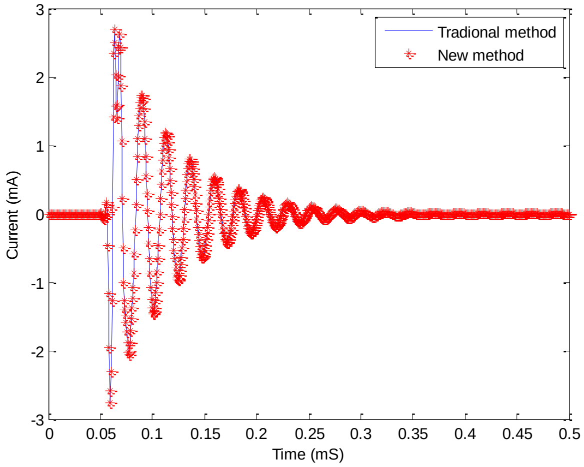

Figure 3. The current at the center of the wire.

Table 1. Comparison in memory usage and computation time.

Method	Time (sec)	Memory (MB)
Traditional MoM-PO hybrid method	1125	5
Proposed MoM-PO hybrid method	361	5

VI. CONCLUSIONS

A new method to compute the time domain hybrid MoM-PO problem is presented. This hybrid method is computationally efficient as it doesn't require iterations between MoM and PO regions for the numerical convergence of the results. In all these results, a T-pulse which is strictly time-limited with most of its energy concentrated in a narrow band is used for excitation. Sample numerical result was presented to illustrate the versatility of this method.

REFERENCES

[1]. R. F. Harrington, *Field Computation by Moment Method*, New York, IEEE Press, 1993.

[2]. B. H. Jung, T. K. Sarkar, Y. Zhang, etc., *Time and Frequency Domain*

Solutions of EM Problems Using Integral Equations and a Hybrid Methodology, New York, IEEE Press, 2010

[3]. D. P. Bouche, F. A. Molinet, and R. Mittra, “Asymptotic and hybrid techniques for electromagnetic scattering,” *Proc. IEEE*, vol. 81, no. 12, pp. 1658-1684, Dec. 1993.

[4]. Bretones, A.R.; Mittra, R.; Martin, R.G., "A hybrid technique combining the method of moments in the time domain and FDTD," *Microwave and Guided Wave Letters, IEEE* , vol.8, no.8, pp.281,283, Aug 1998

[5]. Monorchio, A.; Bretones, A.R.; Mittra, R.; Manara, G.; Martin, R.G., "A hybrid time-domain technique that combines the finite element, finite difference and method of moment techniques to solve complex electromagnetic problems," *Antennas and Propagation, IEEE Transactions on* , vol.52, no.10, pp.2666,2674, Oct. 2004

[6]. G. A. Thiele, “Overview of selected hybrid methods in radiating system analysis,” *Proc. IEEE*, vol. 80, no. I, pp. 67-78, Jan. 1992.

[7]. Z. Mei, *Solving time domain scattering and radiation problems in a marching-on-in-degree technique for method of moment*, Ph.D. thesis, Syracuse. Univ., Syracuse, NY, 2014.

[8]. Z. Mei, T. K. Sarkar, Y. Zhang, and M. Salazar-Palma, “A hybrid method of moment (MoM) and physical optics (PO) technique in the time domain,” *Antennas and Propagation Society International Symposium (APSURSI), 2012 IEEE* , vol., no., pp.1,2, 8-14 July 2012

[9]. Z. Ji, T. K. Sarkar, B. H. Jung, Y.-S. Chung, M. Salazar-Palma, and M. Yuan, “A Stable Solution of Time Domain Electric Field Integral Equation for Thin-Wire Antennas Using the Laguerre Polynomials*,”* *IEEE Trans. Antennas Propag.*, Vol. 52, No. 10, pp. 2641–2649, Oct. 2004.

[10]. Z. Mei, Z. Yu, T.K. Sarkar, B. H. Jung, A. Garcia-Lamperez, and M. Salazar-Palma, “An Improved Marching-on-in-Degree Method Using a New Temporal Basis,” *IEEE Trans. Antennas Propag.*, Vol. 59, No. 12, pp. 4643–4650, Jan. 2011.

[11]. van 't Wout, E.; van der Heul, D.R.; van der Ven, H.; Vuik, C., "Design of Temporal Basis Functions for Time Domain Integral Equation Methods With Predefined Accuracy and Smoothness," *Antennas and Propagation, IEEE Transactions on* , vol.61, no.1, pp.271,280, Jan. 2013

[12]. Z. Mei, Y. Zhang, X. Zhao, B. H. Jung, T.K. Sarkar, and M. Salazar-Palma, “Choice of the scaling factor in a marching-on-in-degree time domain technique based on the associated laguerre functions,” *IEEE Trans. Antennas Propag.*, vol.60, no.9, pp.4463,4467, Sept. 2012.

[13]. Sekeljic, N.J.; Ilic, M.M; Notaros, B.M, "Spatially Large-Domain and Temporally Entire-Domain Electric-Field Integral Equation Method of Moments for 3-D Scattering Analysis in Time Domain," *Antennas and Propagation, IEEE Transactions on* , vol.PP, no.99, pp.1,1.

[14]. Karami, H.; Moini, R.; Sadeghi, S.H.H.; Maftooli, H.; Mattes, M.; Mosig, J.R., "Efficient Analysis of Shielding Effectiveness of Metallic Rectangular Enclosures Using Unconditionally Stable Time-Domain Integral Equations," *Electromagnetic Compatibility, IEEE Transactions on* , vol.56, no.6, pp.1412,1419, Dec. 2014

[15]. Huan Huan Zhang; Zhen Hong Fan; Ru Shan Chen, "Marching-on-in-Degree Solver of Time-Domain Finite Element-Boundary Integral Method for Transient Electromagnetic Analysis," *Antennas and Propagation, IEEE Transactions on* , vol.62, no.1, pp.319,326, Jan. 2014

[16]. Z. Mei, B.-H. Jung, Y. Zhang, X.-W. Zhao, T. K. Sarkar, and M. Salazar-Palma, "A Study on the Numerical Accuracy of the Matrix Elements in a Time Domain MOD Methodology." *Progress In Electromagnetics Research M*, vol. 33, pp.185-196, 2013

[17]. Z. Mei, Y. Zhang, and T. K. Sarkar, "Solving time domain EFIE using higher order basis functions and marching-on in degree method," *Antennas and Propagation Society International Symposium, 2009. APSURSI '09.* IEEE, pp.1,4, 1-5 June 2009.

[18]. Y. Hua and T. K. Sarkar, "Design of Optimum Discrete Finite Duration Orthogonal Nyguist Signals," *IEEE Trans. on Acoustics, Speech & Signal Processing*, Vol. 36, No. 4, pp. 606-608, Apr. 1988.

[19]. T. K. Sarkar, M. Salazar-Palma, and M. C. Wicks, *Wavelet Applications in Engineering Electromagnetics*, Artech House, Norwood, Mass. 2002.

[20]. Z. Mei, T. K. Sarkar, and M. Salazar-Palma, “The design of an ultrawideband T-pulse with a linear phase fitting the FCC mask,” *IEEE Trans. Antennas Propag.*, vol.59, no.4, pp.1432,1436, April 2011.

[21]. Z. Mei, T.K.Sarkar, and M. Salazar-Palma, “A study of wideband pulse shape distortion due to presence of obstacles,” *Microw. Opt. Technol. Lett.*, vol. 55: pp. 1618–1622,2013.

[22]. S. H. Yeung, Z. Mei, T. K. Sarkar, and M. Salazar-Palma, “An ultrawideband T-pulse fitting the FCC mask using a multiobjective genetic algorithm,” *Microwave and Wireless Components Letters, IEEE*, vol.22, no.12, pp.615,617, Dec. 2012

Zicong Mei received BS degree from University of Science and Technology of China (Anhui, China) in 2008, and MS and Ph.D. degree from Syracuse University in 2013 and 2014, respectively. He joined Lorentz Solution, Inc. as a R&D engineer in 2013.

He developed two time domain MoM software TDIE and TFDSIE. He also authored three chapters in a book, namely Time and Frequency Domain Solutions of EM Problems Using Integral Equations and a Hybrid Methodology (Wiley-IEEE, 2010), as well as 14 journal articles and 8 conference papers.